VERTICAL CELLS DRIVEN BY VORTICES - A POSSIBLE MECHANISM FOR THE PRECONDITIONING OF OPEN-OCEAN DEEP CONVECTION

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ABSTRACT

The occurrence of open-ocean deep convection requires a background cyclonic circulation and a preconditioning. Both conditions reduce the stratification of the water column within the cyclonic gyre, which will then become eligible for convection if the surface forcing is sufficiently intense. Therefore, for open-ocean deep convection the generation of vertical cells inside the vortex by any mechanism (not necessary by pure thermodynamical processes) is ultimately important. There are two dynamical mechanisms for inducing vertical cells inside a stably stratified vortex: baroclinic and centrifugal instabilities. The combination of the two is called symmetric instability. In order to verify the importance of symmetric instability on the generation of vertical cells inside the vortex, a tangential velocity field with Gaussian distribution in both radial and vertical under stable stratification is taken as a mean flow field. The disturbances produced from this mean flow are assumed to be two dimensional (no azimuthal dependency) and described by a streamfunction in the radial-vertical sections. This streamfunction satisfies a second-order partial differential equation. The numerical solutions show the generation of vertical cells inside the vortex. The strength and structure of these cells largely depend on the four parameters: Burger number $Bu = (NH/fR)^2$, Rossby number $Ro = V/fR$, barotropic index m_1, and baroclinic index m_2. The larger the values of Ro, m_1, and m_2, or the smaller the value of Bu (weaker stratification for a given size of vortex), the stronger the vertical circulation. The time rate change of density (density redistribution) generated by a vortex and horizontally averaged inside the vortex, indicates the decrease of density in the lower part of the vortex, and the increase of density in the upper part of the vortex. This process, on the time scale of weeks, decreases the static stability and serves as one contributor to preconditioning for the open ocean deep convection.

INTRODUCTION

The association of deep convection with strong vortices has long been observed. Gordon (1978) reported the occurrence of deep convection in February 1977 within the central region of the Weddell Gyre west of Maud Rise. A column of water (14 km radius) was observed in which the normal Antarctic stratification of temperature was absent. The column appeared as a warm, low-salinity surface layer occupying approximately the upper 190 m above a cold nearly homogeneous water column, with a cyclonic eddy (surface velocity greater than 50 cm/s) extending to at least 4000 m.

One may ask such questions: is there any relation between the vortex and the deep convection? and what is the relation? The main task of this paper is to answer these questions and to present a possible mechanism for generating vertical cells by a vortex. In a general sense, convection can be defined as vertical overturning (Chu, 1991), no matter whether the process that generated it is thermodynamical or dynamical.

Vortices represent the most energetic components of the meso-scale eddy field in the world oceans. Their large azimuthal swirl speeds, compared with the rate at which they translate, allows them to carry substantial volumes of fluid with them over long periods of time. Rayleigh (1880, 1916) first investigated the instability of rotating structures in non-rotating fluids. He considered a basic swirling flow of an inviscid fluid which moves with angular velocity $\Omega(r)$, an arbitrary function of the distance r from the axis of rotation. By a simple physical argument Rayleigh then derived his celebrated criterion for stability. Rayleigh's circulation criterion states that a necessary and sufficient condition for stability to axisymmetric disturbances is that the square of the angular momentum does not decrease radially anywhere.

$$C(r) = \frac{1}{r^3}\frac{d}{dr}(r^2\Omega)^2 \tag{1}$$

where the function, $C(r)$, is called the Rayleigh Discriminant. The vortex can be either centrifugally stable or unstable, depending upon the sign of the Rayleigh Discriminant, C. Rayleigh further observed that there is an analogy between the stability of rotating flows and the stability of a stratified fluid at rest in a gravitational field, the analogue of C in fact being the square of the Brunt-Vaisaila frequency.

In subsequent sections a dynamical system is established to discuss the generation of vertical cells inside vortices. The sea water is assumed to be Boussinesq and inviscid. Therefore, the system excludes the damping effect of frictional forces on the vortex. Furthermore, for the sake of simplicity, we also assume no vertical velocity at the top and the bottom of the vortex and no interaction between the vortex and the ambient flow.

2. DYNAMICAL SYSTEM

2.1 Total Flow

A cylindrical coordinate system (r, λ, z) is used in this study with the rotating axis of the vortex as the center of the system (Fig.1). The vertical coordinate is positive upward and zero at the bottom of the vortex.

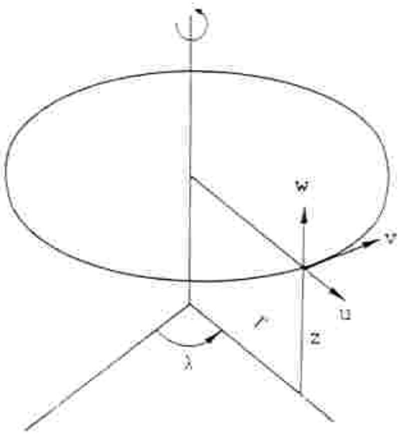

Fig. 1. The cylindrical coordinates (r, λ, z) with the vertical axis at the center of the vortex.

The Boussinesq approximation is assumed for our system. ρ_0 is the characteristic density of the sea water. The total density and pressure can be decomposed into

$$\rho = \rho_0 + \rho_* \tag{2a}$$

$$p = (p_a - \rho_0 g z) + p_* \tag{2b}$$

where p_a is the air pressure at the ocean surface, p_* is the dynamical pressure, and ρ_* is the density deviation from the characteristic value. Under the assumption that the motion is axisymmetric and inviscid, the governing equations are written in the cylindrical coordinates as

$$\frac{du}{dt} - (f + \frac{v}{r})v = -\frac{1}{\rho_0}\frac{\partial p_*}{\partial r} \tag{3a}$$

$$\frac{dv}{dt} + (f + \frac{v}{r})u = 0 \tag{3b}$$

$$\frac{dw}{dt} = -\frac{\rho_*}{\rho_0} g - \frac{1}{\rho_0}\frac{\partial p_*}{\partial z} \tag{3c}$$

$$\frac{\partial (ru)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0 \tag{3d}$$

$$\frac{d\rho_*}{dt} = Q \tag{3e}$$

where u, v, w are radial, tangential, and vertical components of the flow field, respectively. Q is the thermally induced rate of change of density. However, the dynamically driven vertical cells and their contribution to the deep convection are the main items; therefore, Q is set to be zero in this research. The operator d/dt is defined by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + v\frac{\partial}{\partial \theta} + w\frac{\partial}{\partial z} \tag{4}$$

2.2 Basic Flow

A steady axisymmetric and gradiently balanced circular vortex with depth H and radius R, is taken to be the basic flow, i.e.,

$$\bar{u} = 0, \quad \bar{v} = \bar{v}(r, z), \quad \bar{w} = 0, \quad \text{for } 0 < r < R, \ 0 < z < H \tag{5}$$

The momentum equations are

$$(f + \frac{\bar{v}}{r})\bar{v} = \frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial r} \tag{6a}$$

$$-\frac{\bar{\rho}}{\rho_0} g - \frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial z} = 0 \tag{6b}$$

where $\bar{v}$ is the mean tangential current. f is the Coriolis parameter. The differentiation of (6a) with respect to z minus the differentiation of (6b) with respect to r, leads to the thermal wind relation for the basic flow

$$(f + \frac{2\bar{v}}{r})\frac{\partial \bar{v}}{\partial z} = -\frac{g}{\rho_0}\frac{\partial \bar{\rho}}{\partial r} \tag{7a}$$

Therefore, the basic flow is the gradient balanced flow. Including the earth rotation, the angular velocity of water parcels at a radius of r is

$$\Omega = \frac{f}{2} + \frac{\bar{v}}{r}$$

The square of the angular momentum is

$$M = (\Omega r^2)^2 = (\frac{f}{2} + \frac{\bar{v}}{r})^2 r^4$$

The thermal wind relation (7a) becomes

$$\frac{1}{r^3}\frac{\partial M}{\partial z} = \frac{g}{\rho_0}\frac{\partial \bar{\rho}}{\partial r} \tag{7b}$$

2.3 Disturbances

Suppose that axisymmetric disturbances are generated from the basic flow. The total flow field is then a combination of the two parts: basic flow and disturbances.

$$u = u',\quad v = \bar{v} + v',\quad w = w',\quad \rho_* = \bar{\rho} + \rho',\quad p_* = \bar{p} + p' \tag{8}$$

Substituting (8) into (3a-c) and taking into account (6a,b), and assuming small amplitudes of disturbances, a set of equations for the disturbances is then obtained

$$\frac{\partial u'}{\partial t} - (f + \frac{2\bar{v}}{r})v' = -\frac{1}{\rho_0}\frac{\partial p'}{\partial r} \tag{9a}$$

$$\frac{\partial v'}{\partial t} + (f + \frac{\bar{v}}{r} + \frac{\partial \bar{v}}{\partial r})u' + \frac{\partial \bar{v}}{\partial z}w' = 0 \tag{9b}$$

$$\frac{\partial w'}{\partial t} = -\frac{\rho'}{\rho_0}g - \frac{1}{\rho_0}\frac{\partial p'}{\partial z} \tag{9c}$$

$$\frac{\partial (ru')}{\partial r} + \frac{\partial (rw')}{\partial z} = 0 \tag{9d}$$

$$\frac{\partial \rho'}{\partial t} + \frac{\partial \bar{\rho}}{\partial r}u' + \frac{\partial \bar{\rho}}{\partial z}w' = 0 \tag{9e}$$

The vertical circulation generated inside the vortex can be depicted by the streamfunction

$$u' = \frac{1}{r}\frac{\partial \psi}{\partial z}, \quad w' = -\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{10}$$

Eliminating v', p', ρ' from (9a-e), we obtain an equation for the streamfunction ψ,

$$\frac{\partial^2}{\partial t^2}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 \psi}{\partial z^2}\right] + \frac{\partial}{\partial r}\left(\frac{A}{r}\frac{\partial \psi}{\partial r} + \frac{B}{r}\frac{\partial \psi}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{C}{r}\frac{\partial \psi}{\partial z} + \frac{B}{r}\frac{\partial \psi}{\partial r}\right) = 0 \tag{11}$$

which is the same equation used by Ooyama (1966) and Charney (1973), and where

$$A \equiv -\frac{g}{\rho_0}\frac{\partial \bar{\rho}}{\partial z} \tag{12a}$$

$$B \equiv -\frac{1}{r^3}\frac{\partial M}{\partial z} = -\left(f + \frac{2\bar{v}}{r}\right)\frac{\partial \bar{v}}{\partial z} = \frac{g}{\rho_0}\frac{\partial \bar{\rho}}{\partial r} \tag{12b}$$

$$C \equiv \frac{1}{r^3}\frac{\partial M}{\partial r} = \left(f + \frac{2\bar{v}}{r}\right)\left(f + \frac{\bar{v}}{r} + \frac{\partial \bar{v}}{\partial r}\right) \tag{12c}$$

represent three different frequencies: $\sqrt{A}$ is the buoyancy frequency, $\sqrt{B}$ is the baroclinic frequency, and $\sqrt{C}$ is the generalized inertial frequency. Here, the mean density $\bar{\rho}$ is a function of r and z. From (12a) and (12b) we can get the relationship between A and B:

$$\frac{\partial A}{\partial r} = -\frac{\partial B}{\partial z} = \frac{1}{r^3}\frac{\partial^2 M}{\partial z^2} \tag{12d}$$

If the ocean has constant stratification outside the vortex, the integration of (12d) with respect to r leads to

$$A = N^2 + D \tag{13a}$$

$$D = \int_r^R \frac{\partial B}{\partial z}\,dr = -\int_r^R \left[\left(f + \frac{2\bar{v}}{r}\right)\frac{\partial^2 \bar{v}}{\partial z^2} + \frac{2}{r}\left(\frac{\partial \bar{v}}{\partial z}\right)^2\right]dr \tag{13b}$$

where N is the characteristic value of the Brunt-Vaisala frequency, which is a constant in this research. The time scale for deep convection is generally much longer than N^{-1} and f^{-1}, i.e.,

$$|\frac{\partial^2}{\partial t^2}| \ll N^2, \quad |\frac{\partial^2}{\partial t^2}| \ll f^2$$

The time rate of change should be neglected against both the buoyancy frequency and the inertial frequency. Therefore, the equation for streamfunction (11) is simplified into

$$\frac{\partial}{\partial r}(\frac{A}{r}\frac{\partial \psi}{\partial r} + \frac{B}{r}\frac{\partial \psi}{\partial z}) + \frac{\partial}{\partial z}(\frac{C}{r}\frac{\partial \psi}{\partial z} + \frac{B}{r}\frac{\partial \psi}{\partial r}) = 0 \tag{14}$$

which shows the vertical cells in (r, z) section generated by the combined effect of buoyancy, baroclinicity, and inertia of a vortex, which is called the symmetric instability. Here, we are interested in the intrinsic physical mechanisms generating vertical cells inside a vortex. Interaction between the vortex and the ambient flow is beyond the scope of this research. Therefore, it is reasonable to assume that no flow can across the edge of the vortex. The radial velocity at the center of the vortex should be zero. Furthermore, we also assume no vertical velocity at the top and the bottom of the vortex for the sake of simplicity. Thus, the boundary conditions for (14) are

$$\psi = 0, \quad \text{at } r = 0, \ r = R, \ z = 0, \ z = H \tag{15}$$

3. NONDIMENSIONALIZATION

If the independent variables (r, z), mean tangential velocity, squares of the three frequencies (B, C, D), and the streamfunction are nondimensionalized by

$$(r, z, \bar{v}) = (R\tilde{r}, H\tilde{z}, V\tilde{v}), \quad (B, C, D) = f^2(Ro\frac{R}{H}\tilde{B}, \tilde{C}, Ro\frac{R^2}{H^2}\tilde{D}), \quad \psi = VH^2\tilde{\psi} \tag{16}$$

where V is the characteristic tangential velocity of the vortex, and

$$\tilde{B} = -(1 + 2Ro\frac{\tilde{v}}{\tilde{r}})\frac{\partial \tilde{v}}{\partial \tilde{z}} \tag{17a}$$

$$\tilde{C} = (1 + 2Ro\frac{\tilde{v}}{\tilde{r}})(1 + Ro\frac{\tilde{v}}{\tilde{r}} + Ro\frac{\partial \tilde{v}}{\partial \tilde{r}}) \tag{17b}$$

$$\tilde{D} = -\int_{\tilde{r}}^{1}[(1 + 2Ro\frac{\tilde{v}}{\tilde{r}})\frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} + 2\frac{Ro}{\tilde{r}}(\frac{\partial \tilde{v}}{\partial \tilde{z}})^2]d\tilde{r} \tag{17c}$$

where $Ro = V/fR$ is the Rossby number. The nondimensional form of the equation (14) is

$$(Bu + Ro\widetilde{D})\frac{\partial^2\widetilde{\psi}}{\partial\widetilde{r}^2} + 2Ro\widetilde{B}\frac{\partial^2\widetilde{\psi}}{\partial\widetilde{r}\partial\widetilde{z}} + \widetilde{C}\frac{\partial^2\widetilde{\psi}}{\partial\widetilde{z}^2}$$

$$-(Bu + Ro\widetilde{D})\frac{1}{\widetilde{r}}\frac{\partial\widetilde{\psi}}{\partial\widetilde{r}} + \left(\frac{\partial\widetilde{C}}{\partial\widetilde{z}} + Ro\frac{\partial\widetilde{B}}{\partial\widetilde{r}} - Ro\frac{\widetilde{B}}{\widetilde{r}}\right)\frac{\partial\widetilde{\psi}}{\partial\widetilde{z}} = 0 \tag{18}$$

where $Bu = (NH/fR)^2$, is the Burger number, which is the square of the ratio of the Rossby radius of deformation to the radius of the vortex. For a given vortex, the magnitude of Bu indicates the stratification. The smaller the Bu, the weaker the stratification. The nondimensional form of the boundary conditions (15) is

$$\psi = 0, \quad \text{at } \widetilde{r} = 0,\ \widetilde{r} = 1,\ \widetilde{z} = 0,\ \widetilde{z} = 1 \tag{19}$$

4. VERTICAL CELLS DRIVEN BY A GAUSSIAN VORTEX

A cyclonic vortex with Gaussian distribution in both radial and vertical

$$\widetilde{v}(\widetilde{r}, \widetilde{z}) = \exp\left[-m_r(\widetilde{r} - \frac{1}{2})^2 - m_z(\widetilde{z} - \frac{1}{2})^2\right] \tag{20}$$

is chosen as a basic flow in this study, where m_r, m_z are parameters indicating the strength of the shear in radial and vertical directions. Equation (18) with the boundary condition (19) and mean flow (20) is integrated numerically. The successive over-relaxation method is used for the integration. The solutions show the generation of vertical cells inside the vortex even in the relatively strong stratification case ($Bu = 1$). The other three model parameters Ro, m_r, m_z are found to be important in affecting the structure of the vertical cells inside the vortex. For a given vortex size, each parameter, taking only two different values, represents the two different physical conditions, such as, $Ro = 0.1$, for a relatively weak vortex, and $Ro = 0.2$, for a relatively strong vortex; $m_r = 0$, for the absence of radial shear, and $m_r = 4$, for a relatively strong radial shear; and $m_z = 0$, for the absence of vertical shear, and $m_z = 4$, for a relatively strong vertical shear.

Fig.2 shows the streamfunction in a radial-vertical section for $Bu = 1$ and $Ro = 0.1$: (*a*) $m_r = 0$, $m_z = 0$, (*b*) $m_r = 4$, $m_z = 0$, (*c*) $m_r = 0$, $m_z = 4$, (*d*) $m_r = 4$, $m_z = 4$. These cases refer to a relatively strong stratification and a relatively weak vortex.

Several results can be easily drawn from Fig.2:

(a) The vertical cells do exist inside the vortex for all cases, even in the case without radial and vertical shears of the mean tangential velocity as shown in Fig.2a. This strongly suggests that the square of the generalized inertial frequency, $C(r)$, is a fundamental factor in generating vertical circulation inside the vortex (since $B = 0$ and $D = 0$).

(b) Comparison of Fig.2a with Fig.2b shows that without the vertical shear on the mean tangential velocity, the effect of the radial shear ($m_r \neq 0$) on the vertical cells is very small.

(c) Comparison of Fig.2b with Fig.2d indicates that with vertical shear on the mean tangential velocity, the effect of the radial shear on the vertical cells becomes evident. The upper cell penetrates to the base of the vortex, and the two-cell structure becomes a three-cell structure.

(d) Comparison of Fig.2c with Fig.2a leads to the result that the baroclinity always intensifies the strength of the vertical cells.

Fig.3 shows the streamfunction in a radial-vertical section for $Bu = 1$ and $Ro = 0.2$: (*a*) $m_r = 0$, $m_z = 0$, (*b*) $m_r = 4$, $m_z = 0$, (*c*) $m_r = 0$, $m_z = 4$, (*d*) $m_r = 4$, $m_z = 4$. These cases refer to a relatively strong stratification and a relatively strong vortex. Comparison of Fig.3 with Fig.2 shows that the increase of the Rossby number will generally strengthen the vertical cells. Therefore, for a given size of the vortex, the faster the rotation of the vortex, the stronger the vertical cells inside the vortex.

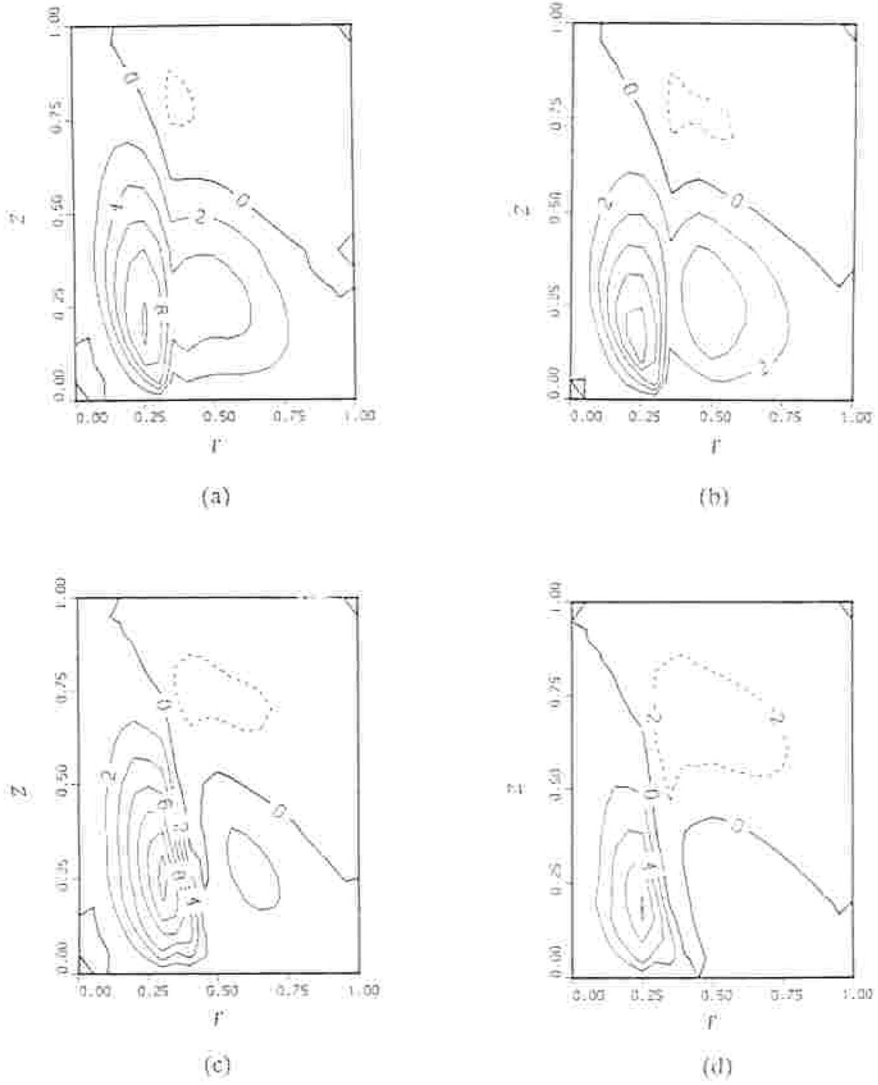

Fig. 2. The streamfunction in radial-vertical section for $Bu = 1$ and $Ro = 0.1$: (a) $m_r = 0$, $m_z = 0$, (b) $m_r = 4$, $m_z = 0$, (c) $m_r = 0$, $m_z = 4$, (d) $m_r = 4$, $m_z = 4$.

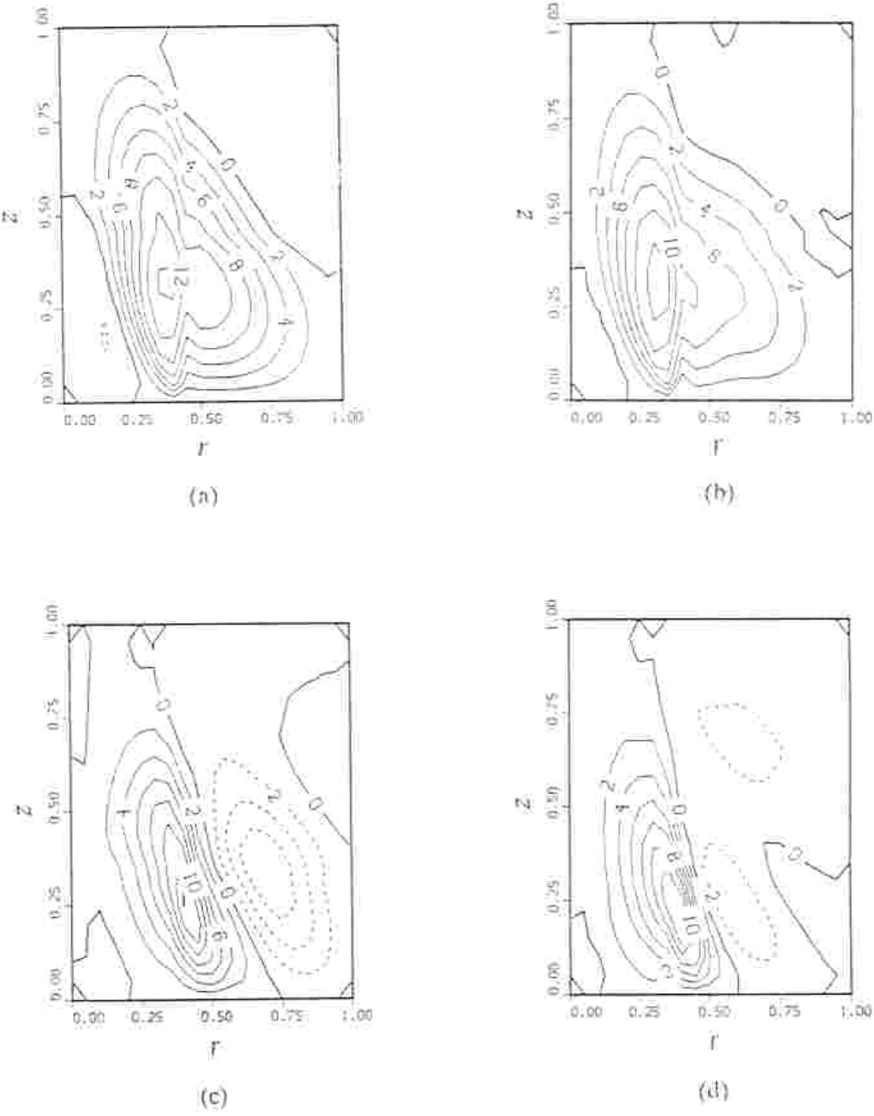

Fig. 3. The streamfunction in radial-vertical section for $Bu = 1$ and $Ro = 0.2$: (a) $m_r = 0$, $m_z = 0$, (b) $m_r = 4$, $m_z = 0$, (c) $m_r = 0$, $m_z = 4$, (d) $m_r = 4$, $m_z = 4$.

5. TIME RATE OF DENSITY INSIDE THE VORTEX

Define a nondimensional density perturbation averaged over the horizontal plane inside the vortex as follows:

$$\sigma = \frac{1}{\pi R^2}\int_0^R 2\pi \frac{\rho'}{\rho_0} r dr = 2\int_0^1 \frac{\rho'}{\rho_0}\tilde{r}\, d\tilde{r} \tag{21}$$

Time t is nondimensionalized by

$$t = (f^{-1})\frac{1}{2Fr^2\delta}\tilde{t} \tag{22}$$

where $Fr = V^2/gH$, is the Froude number, and $\delta = R/H$, is the aspect ratio. Integrating the density equation (9e) over the horizontal plane within the vortex, and using the nondimensional forms of parameters (17a,c), a nondimensional equation of time rate of change of horizontally averaged density is obtained

$$\frac{\partial\sigma}{\partial\tilde{t}} = -\int_0^1 (\tilde{B}\frac{\partial\tilde{\psi}}{\partial\tilde{z}} + \tilde{D}\frac{\partial\tilde{\psi}}{\partial\tilde{r}})d\tilde{r} \tag{23}$$

The derivative, $\partial\sigma/\partial\tilde{t}$, is computed for the case: $Bu = 1$, $Ro = 0.1$, $m_i = 4$, $m_r = 4$ (Fig.2d). The effect of the vertical motion on the density redistribution inside the vortex is shown in Fig.4, where the density decreases in the lower part ($\tilde{z} < 0.27$) of the vortex, and increases in the upper part ($\tilde{z} > 0.27$) of the vortex. This process reduces the stratification and tends to make the density vertically uniform inside the vortex. We view this process as one contribution to the preconditioning of the open ocean deep convection.

6. CONCLUSIONS

(a) This research is intended to bring to the attention of the oceanographic community the importance of symmetric instability in vortex dynamics, and in generating convection. Solutions show generation of a variety of convective cells inside the vortex by this mechanism.

(b) The vertical cells can be generated in the vortex with relatively strong stratification and without any radial or vertical shear. The rotation itself will induce the vertical circulation.

(c) The baroclinicity always intensifies the vertical circulation. However, the effect of the radial shear on the vertical cells becomes important only when it is associated with vertical shear.

(d) The stronger the rotation of the vortex, the stronger the vertical cells inside the vortex.

(e) The time scale for the density redistribution by this dynamically driven overturning is inversely proportional to the square of the Froude number. For numerous observations of vortices in the ocean, this time scale is several weeks. This scale is coincident with the time period needed for the preconditioning.

(f) The solution here refers to the Gaussian type cyclonic vortex. For other kinds of vortices, the solution may vary.

(g) The solution is obtained from assuming rigid boundaries both at the top and the bottom of the vortex. If free boundary conditions or other kinds of boundary conditions are used, the shape of the vertical cells will change, especially near the boundaries, but the basic physics will remain.

7. ACKNOWLEDGEMENTS

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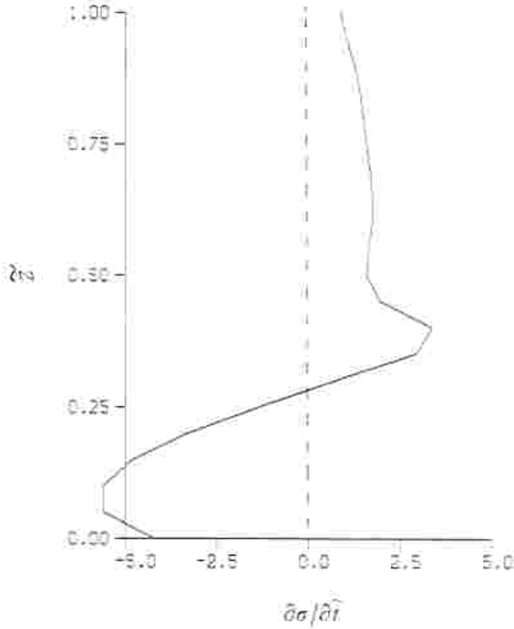

Fig. 4. Vertical dependency of $\partial\sigma/\partial\tilde{t}$ for the vortex with $Bu = 1$, $Ro = 0.1$, $m_r = 4$, and $m_z = 4$.

The time scale for the density redistribution process inside the vortex can be roughly estimated as follows. If the tangential velocity for the vortex V is 50 cm/s, and if the thickness of the vortex H is on the order of 1 km, the square of the Froude number (Fr^2) will be 0.25×10^{-4}. The aspect ratio δ is on the order of 0.1, and the change of the non-dimensional density perturbation $\Delta\sigma$ is on the order of 10^{-3}, and the integration, $I = \int_0^1 (\tilde{B}\partial\tilde{\psi}/\partial\tilde{z} + \tilde{D}\partial\tilde{\psi}/\partial\tilde{r})d\tilde{r}$, is on the order of 5 (Fig. 4). Therefore, the time scale for this density redistribution process is

$$T \sim f^{-1}\frac{\Delta\sigma}{2Fr^2\delta O(I)} \sim 40f^{-1}. \tag{24}$$

which is on the order of few weeks. This scale is coincident with the time period (few weeks) needed for the preconditioning which creates a region of very weak static stability within the vortex (Killworth, 1983).

8. REFERENCES

Charney, J.G., 1973. Planetary fluid dynamics. In: P. Morel (Editor), Dynamical Meteorology. Reidel, Dordrecht, pp. 99-351.

Chu, P.C., 1991. Geophysics of deep convection and deep water formation in oceans. (in this volume).

Killworth, P.D., 1983. Deep convection in the world ocean. Rev. Geophys. & Space Phys., 21: 1-26.

Ooyama, K., 1966. On the stability of the baroclinic circular vortex: a sufficient criterion for instability. J. Atmos. Sci., 23: 43-53.

Rayleigh, Lord, 1880. On the stability, or instability, of certain fluid motion. Scientific Papers, Cambridge University Press, 1: 474-487.

Rayleigh, Lord, 1916. On the dynamics of revolving fluids. Scientific Papers, Cambridge University Press, 6: 432-446.